Coordination of Heterogeneous Multi-agent Systems via Blended Dynamics Theorem

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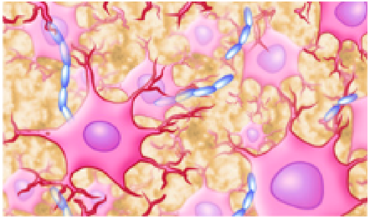
consensus = synchronization : $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \ \forall i, j$

flock of animals



biological organ

networked robots

Q1 How consensus gives rise to emergent behavior?

Q2 How consensus gives rise to robustness of group behavior?

Q3 How the agents perform different tasks in coordination?

Q1. How consensus gives rise to emergent behavior?

emergent behavior $\longleftarrow$ diversity (heterogeneity) + consensus

Consider N agents:

$$\dot{x}_i = \underbrace{f_i(x_i, t)}_{\text{agent vector field}} + \underbrace{u_i(t),}_{\text{signal enforcing consensus}} \qquad i \in \mathcal{N} := \{1, \cdots, N\}$$

Assumption A*: $\exists \theta = [\theta_1, \cdots, \theta_N]^\top$, $\theta_i > 0$, such that

$$\sum_{i=1}^{N} \theta_i = 1 \quad \text{and} \quad \sum_{i=1}^{N} \theta_i u_i(t) = 0, \quad \forall t.$$

Example: $u_i(t) = k \sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t))$ under strongly connected graph

Assumption A* allows decomposition of x_i as

$$x_i = z_o + R_i \tilde{z}, \qquad \forall i \in \mathcal{N}$$

where

$$z_o := \sum_{i=1}^{N} \theta_i x_i, \qquad \tilde{z} = Q^\top \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \qquad \left(\text{i.e., coordinate change: } \begin{bmatrix} z_o \\ \tilde{z} \end{bmatrix} \leftrightarrow \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \right)$$

Multi-agent system in new coordinates:

$$\begin{cases} \dot{z}_o = \sum \theta_i f_i(z_o + R_i \tilde{z}, t) \\ \dot{\tilde{z}} = Q^\top \begin{bmatrix} f_1(z_o + R_i \tilde{z}, t) \\ \vdots \\ f_N(z_o + R_i \tilde{z}, t) \end{bmatrix} + Q^\top \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} \end{cases}$$

If $\tilde{z}(t) \equiv 0$, then $z_o(t)$ is governed by the emergent dynamics:

$$\dot{s} = \sum_{i=1}^{N} \theta_i f_i(s, t) \qquad \text{(blended dynamics)}$$

We need stability of the blended dynamics

In practice, $\tilde{z}(t) \not\equiv 0$, but enforced consensus guarantees

$$\lim_{t\to\infty} \|\tilde{z}(t)\| = 0 \qquad \text{or} \qquad \limsup_{t\to\infty} \|\tilde{z}(t)\| \leq \epsilon$$

In order for $z_o(t)$ of

$$\dot{z}_o = \sum_{i=1}^{N} \theta_i f_i(z_o + R_i \tilde{z}, t)$$

to follow $s(t)$ of

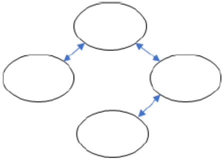
$$\dot{s} = \sum_{i=1}^{N} \theta_i f_i(s, t) \qquad \text{with } s(0) = z_o(0),$$

we require that the blended dynamics is stable.

List of input signals that enforce consensus while satisfying A*

1. Linear coupling
2. Sign coupling
3. PI (Proportional-Integral) coupling
4. Linear coupling in discrete-time
5. Impulsive gossip coupling

For simplicity of today's presentation, let the graph be undirected and connected:



Then, A* holds with $\theta_i = 1/N$, and the blended dynamics has the form of

$$\dot{s} = \frac{1}{N} \sum_{i=1}^{N} f_i(s, t)$$

Blended dynamics theorem: Linear coupling

(JY Kim, Yang, S, & Kim, ECC 2013), (JY Kim, Yang, S, Kim & Seo, TAC 2016)

$$\dot{x}_i = f_i(x_i, t) + k \sum_{j \in \mathcal{N}_i} (x_j - x_i), \qquad x_i \in \mathbb{R}^n,\ i \in \mathcal{N}$$

Theorem: If the blended dynamics

$$\dot{s} = \frac{1}{N} \sum_{i=1}^{N} f_i(s, t) \quad =: \bar{f}(s, t)$$

is contractive[1], then $\forall \epsilon > 0$, $\exists k^*$ such that, with $k > k^*$,

$$\limsup_{t \to \infty} \|x_i(t) - s(t)\| \le \epsilon, \qquad \forall i \in \mathcal{N}.$$

1

$$\exists P > 0 \quad \text{s.t.} \quad P \left(\frac{\partial \bar{f}}{\partial s}(s, t) \right) + \left(\frac{\partial \bar{f}}{\partial s}(s, t) \right)^T P \le -I, \qquad \forall s,\ \forall t.$$

Blended dynamics theorem: Edgewise signum coupling

(JM Seong, PhD thesis, 2024) $\leftrightarrow$ (Franceschelli, Pisano, Giua, & Usai, TAC 2014)

$$\dot{x}_i = f_i(x_i, t) + k \sum_{j \in \mathcal{N}_i} \mathrm{sgn}(x_j - x_i)$$

Theorem: If the blended dynamics

$$\dot{s} = \frac{1}{N} \sum_{i=1}^{N} f_i(s, t)$$

is contractive, then $\exists k^*$ such that, with $k > k^*$,

$$\lim_{t \to \infty} \|x_i(t) - s(t)\| = 0, \qquad \forall i \in \mathcal{N}.$$

Blended dynamics theorem: PI (proportional-integral) coupling

(SJ Lee & S, AUT 2022), (TK Kim, Lee, & S, TAC 2024)

$$\dot{x}_i = f_i(x_i, t) + k \sum_{j \in \mathcal{N}_i} (x_j - x_i) + k \sum_{j \in \mathcal{N}_i} (\xi_j - \xi_i)$$

$$\dot{\xi}_i = -k \sum_{j \in \mathcal{N}_i} (x_j - x_i)$$

Theorem: If the blended dynamics

$$\dot{s} = \frac{1}{N} \sum_{i=1}^{N} f_i(s, t)$$

has an exponentially stable equilibrium s^*, then $\exists k^*$ s.t., with $k > k^*$,

$$\lim_{t \to \infty} \|x_i(t) - s^*\| = 0, \qquad \forall i \in \mathcal{N}.$$

Blended dynamics theorem: Discrete-time case, linear coupling

(JW Kim, Lee, Lee & S, AUT 2024) ← (Wang, Liu, Morse, & Anderson, CDC 2019)

t: discrete-time index

$$x_i[t+1] = \begin{cases} f_i(x_i[t]), & t = 0 \mod k, \\ x_i[t] + \delta \sum_{j \in \mathcal{N}_i} (x_j[t] - x_i[t]), & \text{otherwise} \end{cases}$$

δ: small positive

Theorem: If the blended dynamics:

$$s[t+1] = \frac{1}{N} \sum_{i=1}^{N} f_i(s[t]) \qquad \text{(blended dynamics)}$$

is contractive, then $\forall \epsilon$, $\exists k^*$ s.t. with $k > k^*$,

$$\limsup_{t \to \infty} \|x_i[kt] - s[t]\| \leq \epsilon, \qquad \forall i \in \mathcal{N}.$$

Blended dynamics theorem: Edgewise impulsive gossip coupling

(Tanwani & S, CDC 2021), (Tanwani, S, & Teel, under review)

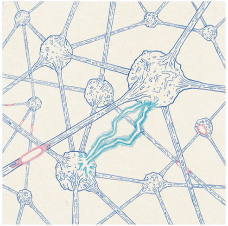
$$\dot{x}_i = f_i(x_i, t) + \sum_{j \in \mathcal{N}_i,\, t_{ij}^* \in \mathcal{T}_{ij}} \frac{(x_j - x_i)}{2} \delta(t - t_{ij}^*)$$

where $\mathcal{T}_{ij}$ is the collection of jump times determined by the timers:

$\dot{\tau}_{ij} = k,\quad \tau_{ij} \in [0,1],$ for every $(i,j) \in \mathcal{E}$

$\tau_{ij}^+ \in [0, \tau_m],\quad \tau_{ij} \in \{1\},\quad \tau_m$: small positive

Image credit: Tom Morris



Theorem: If the blended dynamics

$$\dot{s} = \frac{1}{N} \sum_{i=1}^{N} f_i(s, t)$$

is contractive, then $\forall \epsilon$, $\exists k^*$ such that, with $k > k^*$,

$$\limsup_{t \to \infty} \|x_i(t) - s(t)\| \leq \epsilon, \qquad \forall i \in \mathcal{N}.$$

Blended dynamics theorem: Output coupling

(JG Lee & S, AUT 2020) $\leftarrow$ (Panteley & Loria, TAC 2017)

$$\begin{aligned}\dot{z}_i &= g_i(z_i, x_i, t)\\ \dot{x}_i &= f_i(x_i, z_i, t) + u_i \qquad \rightarrow \text{ e.g. } f_i(x_i, z_i(t), t) + k\textstyle\sum_{j\in\mathcal{N}_i}(x_j - x_i)\end{aligned}$$

For this case, the blended dynamics is:

(blended dynamics) $$\begin{cases}\dot{\hat{z}}_i = g_i(\hat{z}_i, s, t), & i \in \mathcal{N},\\ \dot{s} = \dfrac{1}{N}\displaystyle\sum_{i=1}^{N} f_i(s, \hat{z}_i, t)\end{cases}$$

Other characterizations of the blended dynamics

1. In case of linear coupling

$$\dot{x}_i = f_i(x_i, t) + k \textstyle\sum_{j \in \mathcal{N}_i} (x_j - x_i),$$

as $k \nearrow$, the MAS exhibits two-time-scale behavior and becomes singularly perturbed system:

quasi-steady-state subsystem = blended dynamics.

2. When a MAS is viewed as an input-output system

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} f_1(x_1, t) \\ \vdots \\ f_N(x_N, t) \end{bmatrix} + \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix}, \qquad \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \mathcal{L} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

where $\mathcal{L}$: Laplacian of strongly connected graph,

zero dynamics = blended dynamics.

emergent behavior $\longleftarrow$ heterogeneity + enforced consensus under A*

When a group of heterogeneous dynamics is enforced for consensus under A*, an emergent behavior arises which is governed by the blended dynamics.

Q: What are these agents doing?

$$\dot{x}_1 = -x_1 + 1 + u_1^{\mathsf{PI}}$$
$$\dot{x}_i = 1 + u_i^{\mathsf{PI}}, \quad i = 2, \cdots, N$$

blended dynamics:

$$\dot{s} = \frac{1}{N}(-s + N), \quad s^* = N$$

$\therefore \exists k^* > 0$ s.t., with $k > k^*$,

$$\lim_{t \to \infty} \|x_i(t) - N\| = 0, \quad \forall i$$

Answer:[a] Distributed network size estimator!

♣ This behavior is an emergent one. Also, stability is traded.

[a] (DG Lee, Lee, Kim, & S, CDC 2018)

Idea of designing a multi-agent system or a distributed algorithm:

1. Build the desired dynamics: $\dot{s} = f_{\text{desired}}(s,t)$ (stable in suitable sense)
2. If f_{desired} is decomposed as

$$\dot{s} = f_{\text{desired}}(s,t) = \frac{1}{N}\sum_{i=1}^{N} f_i(s,t)$$

3. Assign each f_i to each agent, and enforce consensus:

$$\dot{x}_i = f_i(x_i,t) + u_i$$

Design a distributed observer for

$$\dot{x} = Ax, \quad y = Cx \in \mathbb{R}^N$$

Desired observer:

$$\dot{s} = As + L(y - Cs) = As + \sum_{i=1}^{N}(L^i y_i - L^i C_i s)$$

Distributed observer:[a]

$$\dot{\hat{x}}_i = A\hat{x}_i + NL^i(y_i - C_i\hat{x}_i) + u_i$$

♣ Even if none of the agents are stable, stability emerges.

[a] (TK Kim, S, & Cho, CDC 2016)

Application to Distributed Optimization

(SJ Lee & S, AUT 2022) cf. (Wang & Elia, Allerton 2010), (Kia, Cortes, & Martinez, AUT 2015), (Hatanaka, Chopra, Ishizaki, & Li, TAC 2018)

$$\min_x \; F(x) = \frac{1}{N}\sum_{i=1}^{N} f_i(x) \qquad \text{where} \qquad F(x)\text{: strongly convex}$$

Gradient descent method:

$$\dot{x} = -\nabla F(x) = -\frac{1}{N}\sum_{i=1}^{N} \nabla f_i(x)$$

Heavy-ball method:

$$\dot{z} = -2\sqrt{\alpha} z - \nabla F(x)$$
$$\dot{x} = z$$

Distributed gradient descent method:

$$\dot{x}_i = -\nabla f_i(x_i) + u_i^{\mathsf{PI}}$$

Distributed heavy-ball method:

$$\dot{z}_i = -2\sqrt{\alpha} z_i - \nabla f_i(x_i)$$
$$\dot{x}_i = z_i + u_i^{\mathsf{PI}}$$

♣ Convexity of f_i is not assumed

Q2. How consensus gives rise to robustness of group behavior?

robustness of group behavior $\longleftarrow$ consensus + large number of agents

Tabareau, Slotine, & Pham, "How Synchronization Protects from Noise," PLOS Comp. Biology, 2010

$$dx_i = f(x_i, t)dt + dW_i + \sum_{j \in \mathcal{N}_i} (x_j - x_i)dt$$

$$z_o := \frac{1}{N}\sum_{i=1}^{N} x_i \quad \rightarrow \quad dz_o = \frac{1}{N}\sum_{i=1}^{N} f(x_i, t)dt + \underbrace{\frac{1}{N}\sum_{i=1}^{N} dW_i}_{= \frac{1}{\sqrt{N}} dW}$$

Blended dynamics gets less sensitive to noise as $N \nearrow$

$\xRightarrow{\text{enforced consensus}}$ Each agent gets less sensitive to noise

We show more robust group behavior

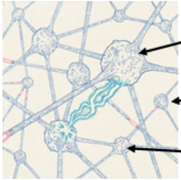
1. against malfunctioning agents
2. against production imperfection

We will also discuss

3. robustness of group behavior against joining/leaving agents

Coupled oscillators

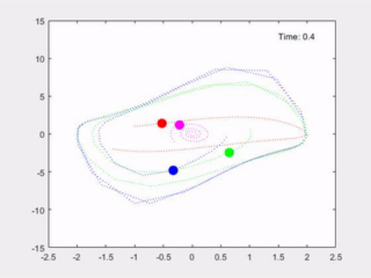
A group of oscillators whose magnitudes, phases, and frequencies are all different (and some agents are not even oscillating) when $u_i \equiv 0$



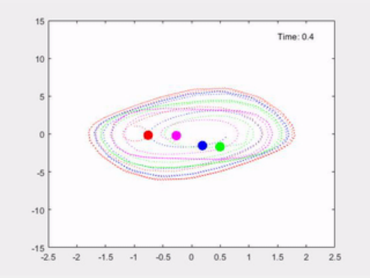
$$\ddot{x}_1 + \mu_1(x_1^2 - 1)\dot{x}_1 + \nu_1 x_1 = u_1$$

$$\ddot{x}_2 + \mu_2(x_2^2 - 1)\dot{x}_2 + \nu_2 x_2 = u_2$$

$$\ddot{x}_3 + \mu_3(x_3^2 - 1)\dot{x}_3 + \nu_3 x_3 = u_3$$

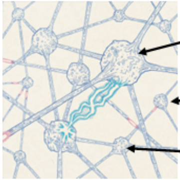


w/o coupling



with coupling

Coupled oscillators (JG Lee & S, CDC 2018)



$$\ddot{x}_1 + \mu_1 (x_1^2 - 1)\dot{x}_1 + \nu_1 x_1 = u_1$$

$$\ddot{x}_2 + \mu_2 (x_2^2 - 1)\dot{x}_2 + \nu_2 x_2 = u_2$$

$$\ddot{x}_3 + \mu_3 (x_3^2 - 1)\dot{x}_3 + \nu_3 x_3 = u_3$$

Theorem: With the output coupling

$$u_i = k \sum_{j \in \mathcal{N}_i} (y_j - y_i) \quad \text{where} \quad y_i = x_i + \dot{x}_i$$

the blended dynamics becomes

$$\ddot{s} + \left(\frac{1}{N} \sum_{i=1}^{N} \mu_i \right) (s^2 - 1)\dot{s} + \left(\frac{1}{N} \sum_{i=1}^{N} \nu_i \right) s = 0.$$

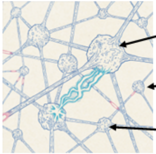
With large k, synchronized oscillation occurs if and only if

$$\frac{1}{N}\sum_{i=1}^{N}\mu_i > 0, \quad \frac{1}{N}\sum_{i=1}^{N}\nu_i > 0, \quad \left[\text{NOT} \quad \mu_i > 0,\ \nu_i > 0,\ \forall i\right]$$

Robustness against malfunctioning agents arises when there are **dominating number of good agents** because the blended dynamics has such property:

$$\dot{s} = \frac{1}{N}\sum_{i=1}^{N} f_i(s) = \frac{1}{N}\Big(\sum_{i\in\mathcal{N}_{\text{good}}} f_i(s) + \sum_{i\in\mathcal{N}_{\text{bad}}} f_i(s)\Big)$$

Robustness against imperfect production



$$\ddot{x}_1 + \mu_1(x_1^2 - 1)\dot{x}_1 + \nu_1 x_1 = u_1$$
$$\ddot{x}_2 + \mu_2(x_2^2 - 1)\dot{x}_2 + \nu_2 x_2 = u_2$$
$$\ddot{x}_3 + \mu_3(x_3^2 - 1)\dot{x}_3 + \nu_3 x_3 = u_3$$

Randomness in cell creation, or imperfect production, makes the parameters μ_i and ν_i random variables, e.g., Gaussian $\mu_i \sim \mathcal{N}(m_\mu, \sigma_\mu^2)$, $\nu_i \sim \mathcal{N}(m_\nu, \sigma_\nu^2)$.

Q: How the multi-agent system becomes **robust against imperfect production**?

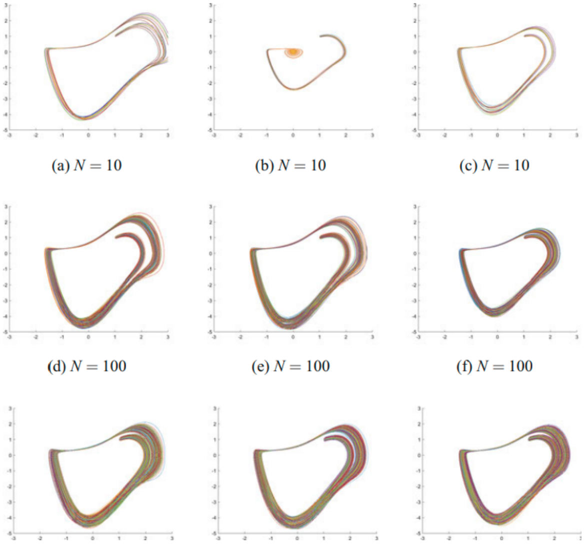
A: By a large number of agents!

$$\frac{1}{N}\sum_{i=1}^{N}\mu_i \sim \mathcal{N}\left(m_\mu, \frac{\sigma_\mu^2}{N}\right), \qquad \frac{1}{N}\sum_{i=1}^{N}\nu_i \sim \mathcal{N}\left(m_\nu, \frac{\sigma_\nu^2}{N}\right)$$

Robustness against imperfect production (JY Kim, Yang, S, Kim & Seo, TAC 2016)

Behavior of randomly generated coupled-oscillators:

# of generation = 3
for each $N = 10, 100, 1000$



(a) $N = 10$ (b) $N = 10$ (c) $N = 10$

(d) $N = 100$ (e) $N = 100$ (f) $N = 100$

(g) $N = 1000$ (h) $N = 1000$ (i) $N = 1000$

Robustness against joining/leaving agents

Example: How to find the average of distributed data $\{a_1, a_2, a_3\}$ in a distributed way?

Using 'average consensus' algorithm

$$\dot{x}_1 = \sum_{j \in \mathcal{N}_1} (x_j - x_1), \quad x_1(0) = a_1$$

$$\dot{x}_2 = \sum_{j \in \mathcal{N}_2} (x_j - x_2), \quad x_2(0) = a_2$$

$$\dot{x}_3 = \sum_{j \in \mathcal{N}_3} (x_j - x_3), \quad x_3(0) = a_3$$

$$x_i(t) \to \frac{a_1 + a_2 + a_3}{3}, \quad \forall i$$

Using 'blended dynamics theorem'

$$\dot{x}_1 = -x_1 + a_1 + u_1^{\mathsf{PI}}$$

$$\dot{x}_2 = -x_2 + a_2 + u_2^{\mathsf{PI}}$$

$$\dot{x}_3 = -x_3 + a_3 + u_3^{\mathsf{PI}}$$

$$\dot{s} = -s + \frac{a_1 + a_2 + a_3}{3}$$

$$s(t) \to \frac{a_1 + a_2 + a_3}{3}$$

$$\therefore \quad x_i(t) \to \frac{a_1 + a_2 + a_3}{3}, \quad \forall i$$

Robustness against joining/leaving agents

The algorithms designed by blended dynamics theorem are "initialization-free" (i.e., it doesn't depend on particular initial conditions). It is because the blended dynamics is stable, by which any initial conditions will be forgotten eventually.

Therefore, if a new agent joins the network with arbitrary initial condition, or if an agent leaves the network with some amount of internal state, it doesn't cause any trouble and the group behavior is still governed by the (updated) blended dynamics. We call this property as **"plug-and-play ready."**

If you design a distributed algorithm that processes some information in the agents, do not put your information in the initial conditions. Put it in the vector fields.

ex: distributed solvers for least square, median, max/min, p-quantile, mode are available in the literature[2]

[2] (Lee & S, CSL 2020), (Lee, Kim & S, TAC 2020), (Seong, Kim, Lee & S, Access 2021), (Huang, S, Yu & Anderson, submitted)

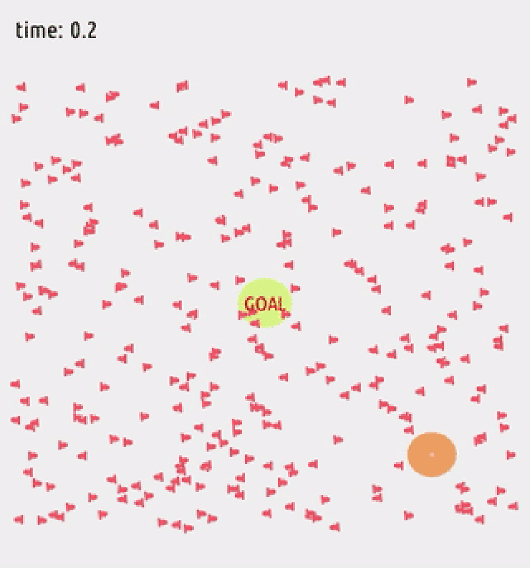
Q3. How different agents perform different tasks in harmony for a global goal?

In order for heterogeneous agents to work in coordination or harmony, there should be some (hidden) internal variables that are in consensus.

Two examples:

1. Multi-channel stabilization of an LTI system
2. Economic power dispatch problem

Example: Identical mobile robots that collaboratively move the object



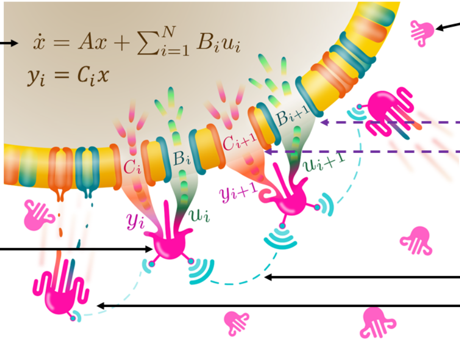
Robots can push or pull the orange object, and the feedback controller of each robot computes the amount of force.

Each controller must be self-organized in coordination of other robots.

Distributed stabilization of multi-channel linear systems[3]

Plant:
to be stabilized
has multi-input-multi-output

Control agent:
can measure y_i and generate u_i
randomly moves around



input channel
output channel

when happen to meet the plant, read (A, B_i, C_i) from the surface, self-organize feedback gains, and start stabilization

communicate with neighbors
can leave any time

Problem: Design identical control agents for stabilization of the plant

[3] a problem extended from (Wang, Fullmer, Liu, & Morse, ACC 2020)

Initial idea: distributed observer + state feedback

First, let us assume

- $\mathcal{N} = \{1, 2, \ldots, N\}$ is fixed, and every agent knows N
- each agent i knows F_i and L_i, which satisfy

$$A + [B_1, \cdots, B_N] \begin{bmatrix} F_1 \\ \vdots \\ F_N \end{bmatrix} \quad \text{and} \quad A + [L_1, \cdots, L_N] \begin{bmatrix} C_1 \\ \vdots \\ C_N \end{bmatrix} \quad \text{are Hurwitz}$$

Controller on each robot is designed by (state feedback + distributed observer):

$$u_i = F_i \hat{x}_i, \qquad \dot{\hat{x}}_i = A\hat{x}_i + N B_i F_i \hat{x}_i + N L_i (C_i \hat{x}_i - y_i) + k \sum_{j \in \mathcal{N}_i} (\hat{x}_j - \hat{x}_i)$$

Idea of distributed design of F_i and L_i (TK Kim, Lee & S, TAC 2024)

Pick some β such that $(A+\beta I)$ is Hurwitz. Solve $X > 0$ for

$$-(A+\beta I)X - X(A+\beta I)^T + 2BB^T = 0.$$

Take F by

$$F = -B^T X^{-1} \quad \left(\text{i.e.,} \quad \begin{bmatrix} F_1 \\ \vdots \\ F_N \end{bmatrix} = -\begin{bmatrix} B_1^T X^{-1} \\ \vdots \\ B_N^T X^{-1} \end{bmatrix}\right).$$

Then, $(A+BF)$ becomes Hurwitz.

Can we solve X in a distributed manner? Yes. Introduce

$$\dot{X}(t) = -(A+\beta I)X(t) - X(t)(A+\beta I)^T + 2BB^T.$$

Then $X(t) \to X$. So, let us compute $X(t)$ in a distributed manner!

Treat

$$\dot{X}(t) = -(A+\beta I)X(t) - X(t)(A+\beta I)^T + 2BB^T$$
$$= -(A+\beta I)X(t) - X(t)(A+\beta I)^T + 2\left(B_1 B_1^T + \cdots + B_N B_N^T\right)$$

as the blended dynamics.

Control agent i runs

$$\dot{X}_i = -(A+\beta I)X_i - X_i(A+\beta I)^T + 2NB_iB_i^T + u_{X_i}^{\mathsf{PI}}, \quad F_i = -B_i^T X_i^{-1}$$

For the observer gain L_i:

$$\dot{Y}_i = -Y_i(A+\beta I) - (A+\beta I)^T Y_i - 2NC_i^T C_i + u_{Y_i}^{\mathsf{PI}}, \quad L_i = Y_i^{-1} C_i^T$$

Proposed (identical) control agent i

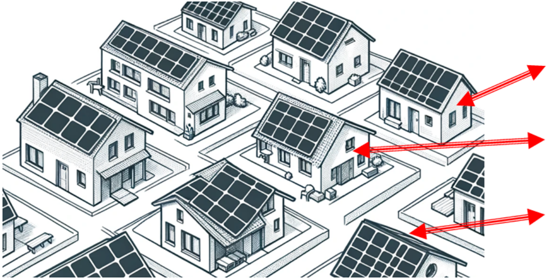
$$
\begin{aligned}
\dot{N}_i &= 1 + u^{\mathsf{PI}}_{N_i} \\
\dot{X}_i &= -(A+\beta I)X_i - X_i(A+\beta I)^T + 2N_i(t)B_iB_i^T + u^{\mathsf{PI}}_{X_i}, \qquad & F_i(t) &= -B_i^T X_i^{-1} \\
\dot{Y}_i &= -Y_i(A+\beta I) - (A+\beta I)^T Y_i - 2N_i(t)C_i^T C_i + u^{\mathsf{PI}}_{Y_i}, \qquad & L_i(t) &= Y_i^{-1} C_i^T \\
\dot{\hat{x}}_i &= A\hat{x}_i + N_i(t)B_iF_i(t)\hat{x}_i + N_i(t)L_i(t)(C_i\hat{x}_i - y_i) + u_{\hat{x}_i}, \qquad & u_i &= F_i(t)\hat{x}_i
\end{aligned}
$$

This achieves

- ♣ distributed operation (no leader, no commander)
- ♣ plug-and-play ready
- ♣ self-organization; F_i and L_i are all different but in coordination for the goal of stabilization; the (hidden) variable to be in consensus is Lyapunov matrices X_i and Y_i (clf)

Economic Power Dispatch Problem for Smart Grid

In an electricity network, each agent i consumes power d_i, and generates power x_i with the generation cost $J_i(x_i)$.



decides d_1 and $J_1(x_1)$
wants to know optimal x_1

decides d_2 and $J_2(x_2)$
wants to know optimal x_2

decides d_3 and $J_3(x_3)$
wants to know optimal x_3

Problem: Find $x_1, \cdots, x_N$ that solve

$$\text{minimize} \quad \sum_{i=1}^{N} J_i(x_i) \qquad \text{subject to} \quad \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} d_i,$$
$$x_i \in \mathcal{X}_i, \quad \forall i = 1, \cdots, N$$

Centralized solution (standard procedure)

1. Dual function

$$g(\lambda) = \inf_{x_i \in \mathcal{X}_i, i \in \mathcal{N}} \Big(\sum_{i=1}^N J_i(x_i) + \lambda \sum_{i=1}^N (x_i - d_i) \Big) = \sum_{i=1}^N \inf_{x_i \in \mathcal{X}_i} \big(J_i(x_i) + \lambda(x_i - d_i) \big) = \sum_{i=1}^N g_i(\lambda)$$

2. Find $\lambda^* = \arg\max g(\lambda)$ by

$$\dot{\lambda} = \nabla g(\lambda) = \nabla g_1(\lambda) + \cdots + \nabla g_N(\lambda)$$

3. Optimal x_i^*:

$$x_i^* = \Phi_i(\lambda^*)$$

Distributed solution (Yun, S & Ahn, AUT 2019)

Each node i runs

$$\dot{\lambda}_i = \nabla g_i(\lambda_i) + u_i^{\mathsf{PI}}, \qquad x_i = \Phi_i(\lambda_i)$$

- ♣ no private information is exchanged
- ♣ initialization-free algorithm (**plug-and-play ready**)
- ♣ not a consensus problem of $x_1, \cdots, x_N$; but, they are in harmony for optimality; the (hidden) variable to be in consensus is the Lagrange multiplier λ_i.

Summary

Q1 How consensus gives rise to emergent behavior?

emergent behavior $\longleftarrow$ heterogeneity + consensus

Q2 How they remain robust against noise/disturbance, malfunctioning agents, production imperfection, joining/leaving agents?

robust emergent behavior $\longleftarrow$ heterogeneity + consensus + many agents

Q3 How different agents perform individual tasks in harmony for global goal?

some (hidden) variable is in consensus

- To design a distributed algorithm or a MAS, thinking of the blended dynamics from the beginning may help!

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